GFR 12 – A

[(See Rule 238 (1))]

UTILIZATION CERTIFICATE (UC) FOR THE YEAR 2021-22

in respect of *RECURRING*

as on 31st March 2022 to be submitted to SERB

Is the UC (Provisional/Audited)

(To be given separately for each financial year ending on 31st March)

Certified that I have satisfied that the conditions on which grants were sanctioned have been duly fulfilled/are being fulfilled and that I have exercised following checks to see that the money has been actually utilized for the purpose for which it was sanctioned:

(i) The main accounts and other subsidiary accounts and registers (including assets registers) are maintained as prescribed in the relevant Act/Rules/Standing instructions (mention the Act/Rules) and have been duly audited by designated auditors. The figures depicted above tally with the audited figures mentioned in financial statements/accounts.

(ii) There exist internal controls for safeguarding public funds/assets, watching outcomes and achievements of physical targets against the financial inputs, ensuring quality in asset creation etc. & the periodic evaluation of internal controls is exercised to ensure their effectiveness.

(iii) To the best of our knowledge and belief, no transactions have been entered that are in violation of relevant Act/Rules/standing instructions and scheme guidelines.

(iv) The responsibilities among the key functionaries for execution of the scheme have been assigned in clear terms and are not general in nature.

(v) The benefits were extended to the intended beneficiaries and only such areas/districts were covered where the scheme was intended to operate.

(vi) The expenditure on various components of the scheme was in the proportions authorized as per the scheme guidelines and terms and conditions of the grants-in-aid.

(vii) It has been ensured that the physical and financial performance under ..... **(CRG/NPDF/ECR.....etc.)** (Name of the scheme has been according to the requirements, as prescribed in the guidelines issued by Govt. of India and the performance/targets achieved statement for the year to which the utilization of the fund resulted in outcomes given at Annexure

I duly enclosed.

(viii) The utilization of the fund resulted in outcomes given at Annexure – II duly enclosed (to be formulated by the Ministry/Department concerned as per their requirements/specifications.)

(ix) Details of various schemes executed by the agency through grants-in-aid received from the same Ministry or from other Ministries is enclosed at Annexure –II (to be formulated by the Ministry/Department concerned as per their requirements/specifications).

Date: 27/04/22
Place:

Haleri 27/04/22 **Signature** **Name:** Prof. Rajib Haleri **Principal Investigator(PI)**	**Signature with Seal** **Name:** **Finance Officer** Finance Officer Tezpur University	27/4/22 **Signature with Seal** **Name:** **Head of Organisation** Registrar Tezpur University

RECURRING
GFR 12 – A
[(See Rule 238 (1))]
UTILIZATION CERTIFICATE (UC) FOR THE YEAR 2021-22
in respect of RECURRING
as on 31st March 2022 to be submitted to SERB
UC (Provisional/Audited)
(To be given separately for each financial year ending on 31st March)

1. Name of the grant receiving Organization : **Tezpur University**
2. Name of Principal Investigator (PI) : **Dr. Rajib Haloi**
3. SERB Sanction order no. & date : MTR/2018/000308 **dated 12th March 2019**
4. Title of the Project : **A Study of controllability of Navier-Stokes Equations with Navier boundary conditions**
5. Name of the SERB Scheme : **MTR – MATRICS**
6. Whether recurring or non-recurring grants : **Recurring**
7. Grants position at the beginning of the financial year (Grants released by SERB)
   (i) Cash in Hand/Bank /Carry forward from previous financial year : 5,596/-
   (Interest Rs. 571+ Carry forward Rs. 5025)
   (ii)Others(3rd year grant) : 2,20,000/-
   (iii)Total : 2,25,596/-

Unspent Balance of Grants received previous years [figure as at Sl.No. 7(iii)]	Interest Earned thereon	Interest deposited back to the SERB	Grants received during the year			Total Available funds (1+2-3+4)	Expenditure incurred	Closing Balances (5-6)	Remark
1	2	3	4			5	6	7	8
			Sanction No. (i)	Date (ii)	Amount (iii)				
5,025/-	**571**	**NIL**	MTR/2018/000308	12 March, 2019	**2,20,000**	**2,25,596**	**2,02,199**	**23,397**	

8. Details of grants received, expenditure incurred and closing balances: (Actuals)

9. Component wise utilization of grants

Grants-in-aid- General	**Total**	**Remark**
Research Grant	1,82,327/-	
Overhead	19,872/-	
GRAND TOTAL	**2,02,199/-**	

10. Details of grants position at the end of the year
    (I) Cash in Hand/Bank : Rs. 23,397/-
    (II) Refunds to SERB, If any : Rs. 23,397/-
    (III) Balance (Carry forward to next financial year): Nil

(CC)

SE for Recurring Grant Scheme: MATRICS Annexure-II

REQUEST FOR ANNUAL INSTALMENT WITH UP-TO-DATE STATEMENT OF EXPENDITURE(SOE)

1. Sanction Order No and date: MTR/2018/000308 **dated 12th March 2019**
2. Total Project Cost: Rs. 6,60,000/-
3. Revised Project Cost: (if applicable) Nil
4. Date of Commencement: 14th March, 2019
5. Statement of Expenditure: Month wise expenditure incurred during current financial year 2021-22.

Month & year	**Expenditure incurred/ committed**	**Remark**
April, 2021	**NIL**	
May, 2021	**NIL**	
June, 2021	**NIL**	
July, 2021	**NIL**	
August, 2021	**1,650.00**	
September, 2021	**NIL**	
October, 2021	**NIL**	
November, 2021	**NIL**	
December, 2021	**NIL**	
January, 2022	**38,178.00**	
February, 2022	**51,558.00**	
March, 2022	**1,10,813.00**	
Total	**2,02,199.00**	

6. Grant received in each year:

Sl No.	Particulars	Amount(in Rs.)
a.	1st Year	2,20,000.00
b.	2nd Year	Nil
c.	3rd year	2,20,000.00
d.	Interest if any	1,904.00 (Rs. 90 for FY 2018-19+Rs. 742 for FY 2019-2020 +Rs. 501 for FY 2020-21+ Rs. 571 for FY 2021-22)
Total(a+b+c+d)		4,41,904.00

SE for Recurring Grant | Scheme: MATRICS | Annexure-III

STATEMENT OF EXPENDITURE(SOE) (1st April 2021 to 31st March 2022)

Sl. No.	Sanctioned Heads	Total fund received till date	Expenditure Incurred				Total Expenditure till 31st March 2022	Balance as on 1st April 2022	Remarks (if any)
			1st Year (14th March 2019 to 31st March 2019)	2nd Year (1st April 2019 to 31st March 2020)	3rd Year (1st April 2020 to 31st March 2021)	4th Year (1st April 2021 to 31st March 2022)			
(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII = IV + V+ VI+VII)	(VIII = III –VII)	
1.	Research Grant	4,00,000/-	-	1,57,758/-	40,042/-	1,82,327/-	3,80,127/-	19,873/-	
2.	Overhead Expenses	40,000/-	-	12,500/-	6,008/-	19,872/-	38,380/-	1,620/-	
3.	Total	4,40,000/-	-	1,70,258/-	46,050/-	2,02,199/-	4,18,507/-	21,493/-	
4.	Interest earned	1,904/-	-	0	0	0	-	1,904/-	Rs. 90 for FY 2018-19+Rs. 742 for FY 2019-2020 +Rs. 501 for FY 2020-21+ Rs. 571 for FY 2021-22
	Grand Total	4,41,904/-	-	1,70,258	46,050/-	2,02,199/-	4,18,507/-	23,397/-	

Signature
Name: Prof. Rajib Halder
Principal Investigator (PI)

Signature with Seal
Name:
Finance Officer

Signature with Seal
Name:
Head of Organisation

Closure Report

File Number : MTR/2018/000308

Project Title : A study of controllability of Navier-Stokes equations with Navier Boundary conditions

Principal Investigator : Dr. Rajib Haloi
Tezpur University
Distt. sonitpur p.b.no.72 napaam, tezpur, Tezpur, Assam-784011

Total Released Amount : 4,40,000 (INR)

Start Date of the Project: 14 Mar, 2019

Date of completion: 13 Mar, 2022 (36 months)

Total Expenditure : 4,18,507 (INR)

Closure Details

Key outcomes or achievements

The Navier-Stokes (N-S) equations arise as mathematical models of important phenomena in fluid mechanics. These equations describe the motion of fluid like water, air, oil under general condition. Let be a smooth bounded and simply connected domain in R^d with d=2 or d=3 and T > 0. Let _T=×(0,T). The N-S equations for imcompressible viscous fluid are defined as u/u+(u.)u+p=uin _T together with the mass conservation equation .u=0and intial value u(0)=u_0defined in ×{0}. Here u is the velocity, p is the pressure. The fundamental no-slip condition of Stokes is defined as u-(u.n)n=0on the boundary ×(0,T), where nis the unit normal to the boundary. The no-slip boundary condition for viscous fluids have zero velocity relative to the boundary. That is the fluid velocity relative to solid boundary is zero. If the slip velocity is on the boundary is assumed to be proportional to the tangential component of the stress, then it is known as slip condition or Navier boundary condition. The degree of the slip is measured by a slip length. This is expressed by the following equations on ×(0,T)as u.n=0and (T(u).n)_t+ku=0where (T(u).n)_t denotes the tangential component of T(u).nand k is the coefficient of proportionality. As a outcome of the study, we obtain the following results: 1. We proved the existence and uniqueness of solutions to the Navier Stokes equations for three-dimensional incompressible fluid in a bounded domain of three dimensions (Published as a paper in Acta Mathematica Scienctia, 39 (6),1628-1638, 2019). 2. The results are further extended to the damped Navier Stokes equations with Navier-Slip boundary conditions for three-dimensional incompressible fluid (Published in Journal of Applied Mathematics and Computing, 66 (1), 307-325, 2021). 3. We also study the approximate controllability of the abstract quasilinear differential equations and published in Electronic Journal of Differential Equations(Vol 2019 (2019) No. 63, 1-14). 4. We also study the feasibility of the applications of numerical scheme for the objective mentioned in the project. As a result, we achieved results in semidiscretization and published in Electronic Journal of Differential Equations(Vol 2020(2020) No. 120, pp 1-10). In addition to the above, we also obtained some inequalities related to the Navier Stokes equations and the works are also published (Bull. Korean Math. Soc. 00 (20XX), No. 0, pp. 1–0 https://doi.org/10.4134/BKMS.b210469).

Detailed research report

A study of controllability of Navier-Stokes equations with Navier Boundary conditions Introduction: The Navier-Stokes (N-S) equations arise as mathematical models of important phenomena in fluid mechanics. These equations describe the motion of fluid like water, air, oil under general condition. Let be a smooth bounded and simply connected domain in R^d with d=2 or d=3 and T > 0. Let _T=×(0,T). The N-S equations for imcompressible viscous fluid are defined as u/u+(u.)u+p=uin _T together with the mass conservation equation .u=0and intial value u(0)=u_0defined in ×{0}. Here u is the velocity, p is the pressure. The fundamental no-slip condition of Stokes is defined as u-(u.n)n=0on the boundary ×(0,T), where nis the unit normal to the boundary. The no-slip boundary condition for viscous fluids have zero velocity relative to the boundary. That is the fluid velocity relative to solid boundary is zero. If the slip velocity is on the boundary is assumed to be proportional to the tangential component of the stress, then it is known as slip condition or Navier boundary condition. The degree of the slip is measured by a slip length. This is expressed by the following equations on ×(0,T)as u.n=0and (T(u).n)_t+ku=0where (T(u).n)_t denotes the tangential component of T(u).nand k is the coefficient of proportionality. Controllability: Controllability is one of the major concept in modern control system. The concept of controllability, introduced by R. Kalman in 1960 refers a mathematical problem which mainly deals with the possibility of forcing the system to a particular state by application of control input. Exact controllability steers a system from the initial state to arbitrary final state while approximate controllability means that the system can be steered to arbitrary small neighbourhood of the final state from the initial state. To study the exact null controllability of solutions to N-S equations, we devide the boundary of the domain two parts on the basis of the application of the control. In one part, the no-slip condition or the Navier condition is prescribed. On the other part we choose a boundary condition which helps to get uniqueness of the solution. In the exact null controllability of the solutions to the N-S equations, we assume that is a non-empty open part of the boundary in×(0,T) which control acts. the fluid flow satisfies u.n=0and (T(u).n)_t+ku=0 on ×(0,T). Let L_^2 () be the closure in L^2 () of smooth divergence free vector fields which are tangent to . Leray proved that u_0L_^2 () there exists a global waek solution to N-S equations with no slip boundary conditions. The question of exact null controllability of N-S equations was posed by J-L Lions [1989] as given T>0 and u_0L_^2 () does there exist a solution to the N-S system with u(.,0)=u_0 such that u(.,T)=0? Objectives: We would like to study the exact null controllability of solutions to the Navier-stokes equation in 3D with a smooth initial data and Navier boundary conditions. the Lagrangian controllability for the perfect fluids. the exact null controllability of solutions to the damped Navier-stokes equation in 2D with a smooth initial data and Navier boundary conditions. the above results of the exact null controllability of solutions to the Navier-stokes equation with the non-linear and nonhomogeneous Navier boundary conditions. As a outcome of the study, we obtain the following results: 1. We prove the existence and uniqueness of solutions of the Navier Stokes equations with Navier slip boundary condition for incompressible fluid in a bounded domain of three dimensions. The results are established by the Galerkin approximation method and improved the existing results. (Reference: Acta Mathematica Scienctia, 39 (6),1628-1638, 2019). 2. The results are further extended to the damped Navier Stokes equations with Navier-Slip boundary conditions for three-dimensional incompressible fluid. The solutions of the damped Navier–Stokes equation with the Navier slip boundary condition in a bounded domain in three dimensions with sufficiently smooth boundary is stdied. We employ the Galerkin method to approximate the solutions of the damped Navier–Stokes equations with the Navier-slip boundary conditions. The existence of the solutions is global for 1. We also established the regularity of the solutions

for 3, and the uniqueness of the solutions for 1 (Reference: Journal of Applied Mathematics and Computing, 66 (1), 307-325, 2021). 3. We 'so study the approximate controllability of the abstract quasilinear differential equations. We study the approximate controllability for quasi linear differential equations with deviating arguments in a Hilbert space. We establish sufficient conditions for the e···istence of a mild solution. We also obtain sufficient conditions for the controllability of quasilinear equations. Further, we use it to establish the approximate controllability for quasilinear differential equations with deviating arguments. We discuss examples in which the analytical results are applied. and published in Electronic Journal of Differential Equations (Vol 2019 (2019) No. 63, 1-14). 4. We also study the feasibility of the applications of numerical scheme for the objective mentioned in the project. We consider a semi-linear differential equation of parabolic type with deviating arguments in a Banach space with uniformly convex dual and apply Rothe's method to establish the existence and uniqueness of a strong solution. We also include an example as an application of the main result (Reference: Electronic Journal of Differential Equations(Vol 2020(2020) No. 120, pp 1-10). In addition to the above, we also obtained some inequalities related to the Navier Stokes equations and the works are also published (Bull. Korean Math. Soc. 00 (20XX), No. 0, pp. 1–0 https://doi.org/10.4134/BKMS.b210469).

N nber of students/Researchers trained : 2

List of Publications (only from SCI indexed journals) :

Title of the Paper	List of Authors	Journal Details	Month & Year	Volume	Status	DOI No	Impact Fact
On solution to the Navier Stokes equations with Navier-Slip boundary conditions for three dimensional incompressible fluid	Pal. S and Haloi, R	ACTA MATHEMATICA SCIENTIA (International)	Sep-2019	39 (1628-1638)	Published	https://doi.org/10.1007/s10473-019-0613-8	1.68
Weighted Integral Inequalities For Modified Integral Hardy Operators, Bull. Korean Math. Soc., https://doi.org/10.4134/BKMS.b210469 pISSN: 1015-8634 / eISSN: 2234-3016 (2022).	Chutia, D. and Haloi,R.	Bulletin of the Korean Mathematical Society (International)	Mar-2022	00 (24)	Published	https://doi.org/10.4134/BKMS.b210469	.5
Rothe's method for solving semilinear differential equations with deviating arguments, Electronic Journal of Differential Equations 2020 (120), 1-10.	D Devi, D Chutia, R Haloi,	Electronic Journal of Differential Equations (International)	Dec-2020	2020 (1-10)	Published	02	.5
Existence of solutions to a non-autonomous abstract neutral differential equation with deviated argument, Electronic Journal of Differential Equations, Vol 2019 (2019) No. 63, 1-14.	Chutia, D and Haloi,R,	Electronic Journal of Differential Equations (International)	Apr-2019	63 (1-14)	Published	01	.49
Existence and uniqueness of solutions to the damped Navier–Stokes equations with Navier boundary conditions for three dimensional incompressible fluid	Pal. S and Haloi, R.	Journal of Applied Mathematics and Computing (International)	Jun-2021	66(1) (307-325)	Published	https://doi.org/10.1007/s12190-020-01437-1	1.686

List of Papers Published in Conference Proceedings, Popular Journals :

Title of the Paper	List of Authors	Journal Details	Month & Year	Volume	Status	DOI No	Impact Factor
Some Existence Results on Impulsive Differential Equations, Proceedings of the Sixth International Conference on Mathematics and Computing, 1262 (2021).	Haloi, R.	Advances in Intelligent Systems and Computing (International)	Sep-2021	1262 (535-548)	Published	https://doi.org/10.1007/978-981-15-8061-1_44	00
Weighted Norm Inequality for General One-Sided Vector Valued Maximal Function, Proceedings of the Sixth International Conference on Mathematics and Computing, 1262 (2021).	Chutia, D., Haloi, R.	Advances in Intelligent Systems and Computing (International)	Sep-2021	1262 (549-563)	Published	https://doi.org/10.1007/978-981-15-8061-1_45	00

List of Patents filed/ to be filed :

Patent Title	Authors	Patent Type	Country Agency Name	Patent Status	Application Grant No.
Not Available					

Any other collaborative work carried out

NA

1. Project Title: A study of controllability of Navier-Stokes equations with Navier Boundary conditions	DST No. **MTR/2018/000308 dated 12th March 2019**
2. PI (Name &address): Dr. Rajib Haloi Department of Mathematical Sciences, Tezpur University, Napaam Pin 784028 Assam	Date of birth: 01.01.1981

3. Broad area of Research:

Navier-Stokes equations with slip boundary conditions.

4. Approved Objectives of the Proposal:

Objectives: We would like to study

1. the exact null controllability of solutions to the Navier-stokes equation in 3D with a smooth initial data and Navier boundary conditions.
2. the Lagrangian controllability for the perfect fluids.
3. the exact null controllability of solutions to the damped Navier-stokes equation in 2D with a smooth initial data and Navier boundary conditions.
4. the above results of the exact null controllability of solutions to the Navier-stokes equation with the non-linear and nonhomogeneous Navier boundary conditions.

Date of Start: 14th March 2019 Date of completion: 13th March 2022	Total cost of Project: 6,60,000.00 Expenditure as on (13th March 22) 6,38,507.00 Capital - General -

6. Methodology: Null and approximate controllability

7.Salient Research Achievements:

7.1 Summary of Progress:

During the project period, we proved the existence and uniqueness of solutions to the Navier Stokes equations for three-dimensional incompressible fluid in a bounded of three dimensions. The results are further extended to the damped Navier Stokes equations with Navier-Slip boundary conditions for three-dimensional incompressible fluid.

We obtain some results in control theory for abstract differential equations and worked with basics of exact null controllability of solutions to the Navier-stokes equation in 3D.

We also establish some results in weighted norm inequality that are used in Navier Stokes equations. Apart from these, we also use some scheme for establishing the existence of strong solution to certain differential equations.

7.2 Research work which remains to be done under the project (for on-going projects):

7.3 List of Publications from this Project (including title, author(s), journals & year(s)

(A) Papers published only in cited Journals (SCI).

1. Chutia, D. and Haloi,R., Weighted Integral Inequalities For Modified Integral Hardy Operators, Bull. Korean Math. Soc., https://doi.org/10.4134/BKMS.b210469 pISSN: 1015-8634 / eISSN: 2234-3016(2022).

2. Pal. S and Haloi, R. Existence and uniqueness of solutions to the damped Navier–Stokes equations with Navier boundary conditions for three dimensional incompressible fluid, Journal of Applied Mathematics and Computing, 66 (1), 307-325, 2021

3. D Devi, D Chutia, R Haloi, Rothe's method for solving semilinear differential equations with deviating arguments, Electronic Journal of Differential Equations 2020 (120), 1-10.

4. Pal. S and Haloi, R. On solution to the Navier Stokes equations with Navier-Slip boundary conditions for three-dimensional incompressible fluid, Acta Mathematica Scienctia, 39 (6),1628-1638, 2019.

5. Chutia, D and Haloi,R, Existence of solutions to a non-autonomous abstract neutral differential equation with deviated argument, Electronic Journal of Differential Equations, Vol 2019 (2019) No. 63, 1-14.

(B) Papers published in Conference Proceedings, Popular Journals etc.

1. Chutia, D. and Haloi, R., Weighted inequalities for one-sided fractional minimal function, The Journal of Analysis, 29, 1151–1163 (2021).

2. Haloi, R. Some Existence Results on Impulsive Differential Equations, Proceedings of the Sixth International Conference on Mathematics and Computing, Advances in Intelligent Systems and Computing, 535-548, 1262 (2021).

3. Chutia, D., Haloi, R., Weighted Norm Inequality for General One-Sided Vector Valued Maximal Function, Proceedings of the Sixth International Conference on Mathematics and Computing, Advances in Intelligent Systems and Computing 549-563, 1262 (2021).

4. Haloi, R., On solutions to abstract Volterra integro-differential equations with iterated deviating arguments, The Journal of Analysis 28 (1), 235-260, 2020.