TEZPUR UNIVERSITY
SEMESTER END EXAMINATION (SPRING)2021
MMS 204: Numerical Analysis

Time: **3 Hours** Total Marks: **70**

The figures in the right-hand margin indicate marks for the individual question.

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1. Identify the correct answer. For each **correct answer 2 marks** will be awarded. **8**

   (a) Let M be the length of the initial interval $[a_0, b_0]$ containing a solution of $f(x) = 0$. Let $\{x_1, x_2, x_3, ...\}$ represent the successive points generated by the bisection method. Then the minimum number of iterations required to guarantee an approximation to the solution with an accuracy of ϵ is given by
   (A) $1 - \frac{\log(\epsilon/M)}{\log 2}$ (B) $1 + \frac{\log(M\epsilon)}{\log 2}$
   (C) $1 + \frac{\log(\epsilon/M)}{\log 2}$ (D) $1 - \frac{\log(\epsilon/2)}{(\log 2)^2}$

   (b) An iterative scheme is given by $x_{n+1} = \frac{1}{5}\left(16 - \frac{12}{x_n}\right)$. Such a scheme with suitable x_0 will
   (A) not converge (B) converge to 1.6
   (C) converge to 1.8 (D) converge to 2

   (c) The Runge-Kutta method of order four is used to solve the differential equation $y' = f(x)$, $y(0) = 0$ with step size h. The solution at $x = h$ is given by
   (A) $y(h) = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right]$
   (B) $y(h) = \frac{h}{6}\left[f(0) + 2f\left(\frac{h}{2}\right) + f(h)\right]$
   (C) $y(h) = \frac{h}{6}\left[f(0) + f(h)\right]$
   (D) $y(h) = \frac{h}{6}\left[2f(0) + f\left(\frac{h}{2}\right) + 2f(h)\right]$

   (d) If $y_{n+1} - y_n = h\phi(x_n, y_n, h)$, $\phi(x, y, h) = af(x, y) + bf(x + h, y + hf)$ is a second order accurate scheme to solve the IVP $y' = f(x, y)$, $y(x_0) = y_0$ then a and b respectively are
   (A) $\frac{h}{2}, \frac{h}{2}$ (B) $\frac{1}{2}, \frac{1}{2}$
   (C) $1, -1$ (D) h, h

2. Define unit round δ. Show that $\delta = \begin{cases} \beta^{-t+1} & \text{for chopped definiton of } fl(x) \\ \frac{1}{2}\beta^{-t+1} & \text{for rounded definiton of } fl(x) \end{cases}$ **2+7**

3. Is the following function $s(x)$ a cubic spline on the interval [1,3]? Is it a natural cubic spline function? **6**

$$s(x) = \begin{cases} x^3 - 3x^2 + 2x + 1 & 1 \le x \le 2 \\ -x^3 + 9x^2 - 22x + 17 & 2 \le x \le 3 \end{cases}.$$

4. Show that Newton-Raphson method has quadratic rate of convergence. **6**

5. Given the equation $x - e^x = 0$ determine the initial approximations for finding the smallest positive root. Use this to find the root correct to three decimal places by using secant method. **2+6**

6. The system of equations

$$\begin{aligned} x^2y + y^3 &= 10 \\ xy^2 - x^2 &= 3 \end{aligned}$$

   has a solution near $x = 0.8$ $y = 2.2$. Perform two iterations of the Newton-Raphson method to obtain this root. **8**

7. Find the value of the integral

$$I = \int_2^3 \frac{\cos(2x)}{1+\sin(x)} dx$$

using Gauss-Legendre two and three point integration rules. **3+3**

8. Find the value of the step length h such that the value of the integration

$$\int_{0.1}^{0.2} \frac{x^2}{\cos x} dx$$

evaluated using trapezoidal rule has an error $< 10^{-6}$. **6**

9. Define linear multistep method. Also define order and error constant. **2+2+2**

10. Use the improved Eulers method to solve the differential equation $y' = x + y^2$ with $y(0) = 1$ in the interval $0 \leq x \leq 0.2$. Take the step size $h = 0.1$. **7**

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