TEZPUR UNIVERSITY

Semester End Examination (Spring) 2021

MMS 203: Ordinary Differential Equation

Time: 3 Hours **Full Marks: 70**

The figures in the right-hand margin indicate marks for the individual question

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1. **Write the integrating fractor of the differential equation**

$$(y^2 + xy + 1)dx + (x^2 + xy + 1)dy = 0.$$

2

2. **Find the wroskian of the functions $e^x \sin x$ and $e^x \cos x$.** **2**

3. **Locate and classify the singular points of the differential equation**

$$(x^2 - x - 2)y'' + (x - 2)y' + xy = 0.$$

3

4. **Compute the indicial equation of $x^2y'' + xy' + (x^2 - \frac{1}{9})y = 0$** **3**

5. **Given that $y_1 = x^3$ is a solution of the differential equation $x^2y'' - 5xy' + 9y = 0$. Find the other linearly independent solution.** **3**

6. **Justify: $0 < x < 3$ is the largest interval in which the solution of the IVP**

$$(x^2 - 3x)y'' + xy' - (x + 3)y = 0, y(1) = 2, y'(1) = 1$$

is certain to exist. **3**

7. **Plot the slope line of the differential equation $y' = xy$ in a region containing the point $(1, 2)$.** **5**

8. **Consider the initial value problem $y' = y^{\frac{1}{3}}, y(0) = 0$. Is there a solution that passes through the point $(1, 1)$? If so, find it.** **5**

9. **Let y_1 and y_2 be two solutions of the differential equations $y'' + p(x)y' + Q(x)y = 0, x \in [a, b]$. Show that the wronskian $W(y_1, y_2)(x)$ of y_1 and y_2 satisfies the following relation**

$$W(y_1, y_2)(x) = ce^{-\int p(x)dx},$$

where c being an arbitrary constant. **5**

10. **Using Laplace transformation solve: $y'' - 2y' + 2y = 0, y(0) = y'(0) = 1$.** **5**

11. **Consider the initial value problem**

$$y' = x^2y - x, y(0) = 0.$$

Determine the Picard's approximation y_n and find the limit of the sequence $\{y_n\}$. **6**

12. **Find the Laplace transform of the following functions:**
$(i)t\cos^2 2t, \quad (ii)e^{2t}\sin 3t \cos 2t$ **8**

13. Find the fundamental matrix of the following system: **6**

$$Y' = AY$$

where

$$A = \begin{bmatrix} 5 & -3 & -2 \\ 8 & -5 & -4 \\ -4 & 3 & 3 \end{bmatrix}$$

14. Using the method of untetermined coefficients find a particular solution of $y''+3y'+y = x^2+3\sin x$. **6**

15. Find power series solution in powers of x **of the equation** $y'' + xy' + (2x^2 + 1)y = 0$ **.** **8**

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