TEZPUR UNIVERSITY
Semester End Examination (Spring) 2021
MMS 201: Complex Analysis

Time: **3 Hours** Total Marks: **70**

The figures in the right-hand margin indicate marks for the individual question.

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(*Attempt maximum of* 70 *marks. Q.*1 *and Q.*2 *are compulsory.*)

1. Choose the **correct answer** from the following questions. **11x2**

(a) Which of the following points lies inside the set $A = \{z \in \mathbb{C} : |z + i/2| < 1\}$?
(i) $-\dfrac{1}{2} + i$ (ii) $\dfrac{1}{2} + i$ (iii) $-\frac{1}{2} - i$ (iv) $-1 - \dfrac{i}{2}$.

(b) The complex number z is such that $|z| = 1,\ z \neq -1$ and $\omega = \frac{z-1}{z+1}$. Then real part of ω is
(i) $\frac{1}{|z+1|^2}$ (ii) $\frac{-1}{|z+1|^2}$ (iii) $\frac{\sqrt{2}}{|z+1|^2}$ (iv) 0.

(c) The $\lim_{z\to 0} \frac{\bar{z}}{z}$
(i) exists and is 1 (ii) does not exists (iii) exists and is -1 (iii) exists along straight lines and are equal.

(d) The value of $\displaystyle\lim_{z\to 1+i} \frac{z^2 - 2i}{z^2 - 2z + 2}$ is
(i) $\dfrac{1-i}{i}$ (ii) $1 + i$ (iii) $1 - i$ (iv) $\dfrac{1+i}{2i}$.

(e) The function $f(z) = \frac{z^3 - z_0^3}{z - z_0}$ is not defined at $z = z_0$. What value must $f(z_0)$ take to ensure continuity of f at $z = z_0$
(i) 0 (ii) 1 (iii) z_0 (iv) $3z_0^2$

(f) The function $f(z) = \frac{xy}{2x^2+y^2}$ when $z \neq 0$ and $f(z) = 0$ is
(i) continuous at $z = 0$ (ii) not continuous at $z = 0$
(iii) differentiable at $z = 0$ (iv) continuous but not differentiable at $z = 0$

(g) Let $f(z) = \sqrt{r}e^{\imath\theta/2},\ (r > 0,\ \alpha < \theta < \alpha + 2\pi)$. Then $f'(z) =$
(i) $\dfrac{1}{2f(z)}$ (ii) $\dfrac{1}{f(z)}$ (iii) $\dfrac{f(z)}{2}$ (iv) does not exist

(h) For a complex function,
(i) it is differentiable implies it satisfies Cauchy-Riemann equations
(ii) it is differentiable implies it may or may not satisfy Cauchy-Riemann equations
(iii) it satisfies Cauchy-Riemann equations implies it is differentiable
(iv) it satisfies Cauchy-Riemann equations implies it is analytic.

(i) If $f(z) = \dfrac{y + ix}{x^2 + y^2},\ z \neq 0$ and $f(0) = 0$, then
(i) f is an entire function (ii) f is analytic at 0
(iii) f is differentiable at 0 (iv) f is not differentiable at 0.

(j) For any simple closed contour C around the origin, $\displaystyle\int_C \frac{dz}{z}$ is
(i) 0 (ii) $2\pi i$ (iii) 0 or $2\pi i$, depending on C
(iv) none of the above

(k) For the function $f(z) = \dfrac{1 - e^{-z}}{z}$, the point $z = 0$ is
(i) an essential singularity (ii) a pole of order 0 (iii) a pole of order 1
(iv) a removable singularity

2. Answer the following questions. **9x2**

(a) The domain of the function $f(z) = 1/(1+z^2)$ is ____________

(b) The function $f(z) = z + 1/z,\ z \neq 0$ in polar form is ____________

(c) The image of the line $\{x = 1\}$ under the function $f(z) = z^2$ is ____________

(d) The $\lim_{z\to 2+3i}[\frac{x^2+5-y^2+2ixy-12i}{x-2+iy-3i}]$ is ____________

(e) The points of discontinuities of $f(z) = \frac{2z-3}{z^2-2z+1}$ are ____________

(f) For $f(z) = \bar{z}$, f is differentiable at ____________

(g) The function $f(z) = z^2$ satisfies the Cauchy-Riemann equations at ____________

(h) A function is said to be analytic at $z = z_0$ if ____________

(i) For two concentric circles C_1 and C_2, $\int_{C_1} f(z)dz = \int_{C_2} f(z)dz\ (\neq 0)$ always, when ____________

3. Show that complex exponential e^z can be bounded for Re $z \leq 0$. **5**

4. Define principal value of log z. Find $log(-1)$ and $Log(-1)$. **2+1+2=5**

5. Evaluate the following integrals: **2+3=5**

(i) $\int_1^2 (\frac{1}{t} - i)^2 dt$.

(ii) $\int_0^{\pi/6} e^{2it} dt$.

6. Find $\int_C \bar{z}dz$, where C is the right half of the unit circle $|z| = 1$ going from $-i$ to i. **5**

7. State and prove the Cauchy's theorem for contour integration. **3+7=10**

8. Let C be the positively oriented circle $|z| = 2$. Evaluate $\int_C \dfrac{zdz}{(z+i)(9-z^2)}$, using Cauchy integral formula. **5**

9. Write the Maclaurin series expansion of $f(z) = \frac{1}{1-z}$ in $|z| < 1$. **5**

10. Write the Laurent series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ about $z = 1$. **5**

11. State Cauchy's residue theorem. Use it to evaluate the integral $\int_C \frac{5z-2}{z(z-1)} dz$ where C is the circle $|z| = 2$. **3+7=10**

12. Using residue show that $\displaystyle\int_0^\infty \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$. **5**

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