MMS/TU/CODL/AUT/2021

Centre for Open and Distance Learning
Tezpur University
MMS 202: TOPOLOGY

Time: hours | Total Marks: 70

Answer as per instructions. The figures in the right-hand margin indicate marks for the individual questions.

1. Choose the correct option/s (write (a), (b), (c) or (d) against question number: more one may be correct) 10

   (i) Boundedness is
   (a) a topological property (b) preserved under continuous functions.
   (c) not a topological property (d) preserved under open functions

   (ii) Which of the following is/are true about product topology:
   (a) Projections are always continuous
   (b) It is finer than the box topology on a finite product.
   (c) If X is Hausdorff, then $X \times X$ need not be Hausdorff.
   (d) It is the finest topology for which projections are continuous.

   (iii) Let $\mathbb{R}_1$ be $\mathbb{R}$ with usual topology and $\mathbb{R}_2$ is $\mathbb{R}$ with lower limit topology. Which of the following mappings is/are NOT continuous:
   (a) $f : \mathbb{R}_1 \to \mathbb{R}_2$: $f(x) = x$ (b) $f : \mathbb{R}_2 \to \mathbb{R}_1$: $f(x) = x$
   (c) $f : \mathbb{R}_1 \to \mathbb{R}_1$: $f(x) = x$ (d) $f : \mathbb{R}_2 \to \mathbb{R}_2$: $f(x) = x$

   (iv) The interior of a set is the
   (a) largest open set containing the set.
   (b) smallest closed set containing the set.
   (c) largest open set contained in the set.
   (d) smallest open set containing the set.

   (v) Components in a topological space are
   (a) Connected and compact
   (b) Compact but not connected
   (c) Connected and closed
   (d) Connected but not closed

2. (a) Show that the union of topologies on a set need not be a topology. 4
   (b) Show that $\{(p, q) : p, q \in \mathbb{Q}\}$ is a basis for the usual topology. 4

3. (a) (i) What is the product topology on the cartesian product of two topological spaces X and Y ? 2
   (ii) Let $f_1 : A \to X$ and $f_2 : A \to Y$ be two functions. Let $f : A \to X \times Y$ be such that $f(x) = (f_1(x), f_2(x))$. Then show that f is continuous iff $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous. 6

4. (a) Let X be first countable at x. Show that for $A \subseteq X$, $x \in \overline{A}$ if and only if there exists a sequence in A converging to x. 4
   (b) Which of the following spaces are second countable (*justify*) : 2+2=4
   (i) $\mathbb{R}$ (with discrete topology) (ii) $\mathbb{R}_\ell$, $\mathbb{R}$ with lower limit topology
   (c) Show that every second countable space contains a countable dense subset. 4

5. Answer the following: 4 × 3=12
   (a) Show that the co-finite topology on an infinite set is T_1 but not Hausdorff.
   (b) Let $f, g : X \to Y$ be continuous. Assume that Y is Hausdorff. Show that $\{x|f(x) = g(x)\}$ is closed in X.
   (c) Show that a convergent sequence in a T_2 (Hausdorff) topological space has a unique limit.

6. (a) Prove the following: :
   (i) The union of connected sets in a connected space is connected if they have a common point. 4
   (ii) The product of any two connected spaces is connected. 4
   (b) Show that a path-connected space is always connected. 2

7. (a) Show that any compact subset of a Hausdorff space is closed. 4
   (b) Show that continuous image of a compact space is compact. 4
   (c) Show that a continuous bijection from a compact space to a Hausdorff space is a homeomorphism. 2

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